<u>Additional Problems</u>

1. Suppose that a client of the ABC brokerage firm seeks to have her order of X = 500,000 shares executed within 3 hours with the best possible execution; that is, she wants to realize the lowest possible total execution cost B. ABC works with a market impact model that estimates the cost of each of n equal-sized executions, B/n to have a fixed component F and a declining variable cost component $v(X^{\cdot}/n)^2$, where X_i/n is the number of shares traded in any given execution i:

$$B/n = F + v(X/n)^2$$

Broker analysts have estimated F to be $10, based on the order processing costs of each transaction. Market impact costs are estimated with the square root function with v equal to .0001.

a. What are the optimal number of equal size transactions and what are the optimal transaction sizes?

b. What are the total market impact or slippage costs associated with the 500,000 share order?

c. Suppose that F in the market impact model is interpreted differently. Rather than representing fixed administrative costs of a transaction, $F = r\cdot n^s$, where coefficients r = .1 and s = .5 were determined as part of a new OLS regression that still determined numerical values for v = .0001 and m = 2. The function F is now considered to reflect the total slippage associated with n transactions for stock per hour. Thus, the market impact model is now estimated as follows:

$$\mathbf{B/n = F + v(X/n)^2 = r\cdot n^s + v(X/n)^2 = .1n^{.5} + .0001(500{,}000/n)^2}$$

Based on this new market impact model, what are the optimal number of equal size transactions and what are the optimal transaction sizes?

2. Sniffing algorithms attempt to discern competitors' algorithms. In some instances, mere observations of competitor trading activity can reveal information as to competitor strategies, latent demand (icebergs or dark pools), etc. Beagle Trading is a proprietary trading company that seeks to sniff out its competitors' trading algorithms in an effort to detect their likely order flows. In an effort to sniff out trade slicing models used by brokers, Beagle intends to rely on the following market impact model estimated from an OLS based on its own trading experience:

$$B/n = F + v(X/n)^2$$

where Beagle estimates the cost of each of n equal-sized executions, B/n to have a fixed component F=10 and a declining variable cost component $v(X^{\cdot}/n)^2$, where v = .0001 and X_i/n is the number of shares traded in any given execution i. Beagle observes a particular broker repeatedly placing buy orders of c = 214 shares of Stock GMNX in various equities marketplaces. Beagle assumes that these order sizes are optimal given its market impact model.

a. Based on the observed order sizes c = X/n = 214, what can Beagle infer about the latent demand for this stock? That is, is Beagle able to sniff out the total number of shares that the broker seeks to purchase based on its market impact model and observed order sizes? Why or why not?

b. Suppose that F in the market impact model is revised to be interpreted and calculated differently. Rather than representing fixed administrative costs of a transaction, F will equal rn^s, where coefficients r = .1 and s = .5 were determined as part of the same OLS that determined numerical values for v = .0001 and m = 2. The function F is now considered to reflect the total slippage associated with n transactions for stock per hour. Thus, this revised market impact model is now estimated as follows:

$$B/n = F + v(X/n)^2 = rn^s + v(X/n)^2 = rn^s + v(X/n)^2 = .1n^{.5} + .0001(X/n)^2$$

Suppose that Beagle observes that a broker has submitted orders of c = 214 shares. Based on these observed order sizes, and with its revised market impact model, what can Beagle infer about the latent demand for this stock? That is, is Beagle now able to sniff out the total number of shares that the broker seeks to purchase based on its market impact model and observed order sizes? Why or why not? If Beagle can infer a latent demand, how many shares does the broker intend to purchase, including transactions already observed?

Solutions

1.a. First, total the transactions costs B and set the derivative of total transaction costs with respect to the number of slices equal to zero and solve for X:

$$B = nF + nv\left(\frac{X}{n}\right)^m = 10n + .0001\frac{500{,}000^2}{n} = 10n + 25{,}000{,}000n^{-1}$$

$$\frac{dB}{dn} = \frac{d\left[nF+nv\left(\frac{X}{n}\right)^m\right]}{dn} = F - (m-1)v\frac{X^m}{n^m} = 10 - \frac{25{,}000{,}000}{n^2} = 0$$

$$n = \sqrt[m]{\frac{v(m-1)X^m}{F}} = \sqrt{2{,}500{,}000} = 1{,}581$$

Thus, we find that the optimal number of slices or transactions is n = 1,581, with X/n, the size of each transaction equal to 500,000/1,581 = 316. One note here: First, non-integer numbers of transactions should probably be prohibited, hence, we have rounded our figures.

b. Total slippage costs equal $nv(X/n)^m = .1581 \cdot 316^2$ = \$15,811.38. Note that fixed administrative transactions costs also total to \$15,811.38 = 1581 · 10 (Again, figures are rounded).

c. First, we write the total slippage cost function as n · B/n as follows:

$$B = nrn^s + nv\left(\frac{X}{n}\right)^m = rn^{s+1} + nv\left(\frac{X}{n}\right)^m$$

$$= .1n^{1.5} + .0001\frac{500{,}000^2}{n} = .1n^{1.5} + 25{,}000{,}000n^{-1}$$

Next, we solve for n, the optimal number of order slices to execute as follows:

$$\frac{dB}{dn} = \frac{d\left[nrn^s+nv\left(\frac{X}{n}\right)^m\right]}{dn} = r(s+1)n^s - (m-1)v\frac{X^m}{n^m} = .15n^{.5} - \frac{25{,}000{,}000}{n^2} = 0$$

Unfortunately, solving for n requires substitution. We substitute and iterate to find that n = 1944. Since X = 500,000, the optimal order size is 500,000/193 = 257 (allow for rounding to obtain integer values).

2.a. Beagle can observe slice sizes only. Based on its market impact model, these individual execution sizes c = 214 = X/n do not reveal either X or n. We need to find n to solve for X. Beagle cannot observe the number of orders n until the purchase program is complete, therefore it cannot infer total or latent demand for the stock. Optimal slice or execution size will not vary with the size of the total demand with the given slippage cost function.

b. Based on its own revised market impact function, Beagle will assume that the total slippage for the stock purchase program is determined from the following:

$$B = nrn^s + nv\left(\frac{X}{n}\right)^m = rn^{s+1} + nv\left(\frac{X}{n}\right)^m = .1n^{1.5} + .0001\frac{X^2}{n} = .1n^{1.5} + .0001X^2n^{-1}$$

Total slippage is minimized when the derivative of the market impact function B is minimized with respect to n. Thus, we will find and set equal to zero the derivative of B with respect to n:

$$\frac{dB}{dn} = \frac{d\left[nrn^s+nv\left(\frac{X}{n}\right)^m\right]}{dn} = r(s+1)n^s - (m-1)v\frac{X^m}{n^m} = .15n^{.5} - .0001 \cdot \frac{X^2}{n^2} = 0$$

Since slices or orders of size 214 shares were observed by Beagle, who will assume that these slices are of optimal size, c = X/n = 214. Now, we can solve for n in the following to obtain the optimal number of orders in this purchase program:

$$.15n^{.5} - .0001 \cdot 214^2 = 0$$

$$n = \left(\frac{\left(\frac{X}{n}\right)^m \cdot v}{r \cdot (1+s)}\right)^{1/s} = \left(\frac{.0001(214)^2}{.15}\right)^2 = 933$$

Answers might vary from 931 to 935 based on rounding to integer values. Thus, 933 slices with 214 shares each indicates a total or latent demand for 933 · 214 = 200,000 shares, again, subject to rounding errors.